Exercise 56

(a) The curve $y = |x|/\sqrt{2 - x^2}$ is called a *bullet-nose curve*. Find an equation of the tangent line to this curve at the point $(1, 1)$.

(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

Solution

A point on the tangent line is known, so all that's needed is its slope. Take a derivative of the given function

$$\begin{aligned} y' = \frac{d}{dx}\left(\frac{x}{\sqrt{2-x^2}}\right) &= \frac{\left[\frac{d}{dx}(x)\right]\sqrt{2-x^2} - \left[\frac{d}{dx}(\sqrt{2-x^2})\right](x)}{2-x^2} \\ &= \frac{(1)\sqrt{2-x^2} - \left[\frac{1}{2}(2-x^2)^{-1/2} \cdot \frac{d}{dx}(2-x^2)\right](x)}{2-x^2} \\ &= \frac{(1)\sqrt{2-x^2} - \left[\frac{1}{2}(2-x^2)^{-1/2} \cdot (-2x)\right](x)}{2-x^2} \\ &= \frac{\sqrt{2-x^2} + \frac{x^2}{\sqrt{2-x^2}}}{2-x^2} \\ &= \frac{\frac{2-x^2}{\sqrt{2-x^2}} + \frac{x^2}{\sqrt{2-x^2}}}{2-x^2} \\ &= \frac{2}{(2-x^2)^{3/2}} \end{aligned}$$

and evaluate it at $x = 1$.

$$y'(1) = 2$$

Therefore, the equation of the tangent line to $y = |x|/\sqrt{2-x^2}$ at $(1, 1)$ is

$$y - 1 = 2(x - 1).$$

Below is a graph showing the function and the tangent line.